

Mathematics Assignment For Class XII

General Directions For Students : Whatever be the notes provided , everything must be copied in the maths copy and then do the homework in the same copy

Chapter 2: Inverse Trigonometric Function (Part -1)

Topic : Introduction (principal value)

- If a function f is one - one and onto , then its inverse exists
- The inverse of f is denoted by ' f^{-1} '
- Inverse trigonometric functions are not one -one over their whole domains
- If we restrict the domain of trigonometric functions , then it becomes bijective and the inverse trigonometric functions are defined within the restricted domains

Domain and range of Inverse Trigonometric Function

	Function	Domain	Range
1	$y = \sin^{-1} x$	$-1 \leq x \leq 1$	$\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$
2	$y = \cos^{-1} x$	$-1 \leq x \leq 1$	$[0, \pi]$
3	$y = \tan^{-1} x$	$-\infty < x < \infty$	$\left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$
4	$y = \cot^{-1} x$	$-\infty < x < \infty$	$(0, \pi)$
5	$y = \operatorname{cosec}^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, 0 \right) \cup \left(0, \frac{\pi}{2} \right]$
6	$y = \sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[0, \frac{\pi}{2} \right) \cup \left(\frac{\pi}{2}, \pi \right]$

Exercise 2.1 Q.1i) Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$$\text{Let } \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = x, \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\Rightarrow \sin x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin x = \sin \frac{\pi}{4}$$

$$\Rightarrow x = \frac{\pi}{4} \Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

Exercise 2.1 Q.3i) Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$

$$\text{Let } \cos^{-1}\left(-\frac{1}{2}\right) = x, \quad 0 \leq x \leq \pi$$

$$\Rightarrow \cos x = -\frac{1}{2}$$

$$\Rightarrow \cos x = \cos(\pi - \frac{\pi}{3}) = \cos \frac{2\pi}{3}$$

$$\Rightarrow x = \frac{2\pi}{3}$$

$$\Rightarrow \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3} \text{ Ans}$$

Exercise 2.1 Q.6ii) Find the principal value of $\tan^{-1}(\tan \frac{9\pi}{8})$.

$$\text{Let } x = \tan^{-1}(\tan \frac{9\pi}{8}), \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$\Rightarrow \tan x = \tan \frac{9\pi}{8} = \tan(\pi + \frac{\pi}{8}) = \tan \frac{\pi}{8}$$

$$\Rightarrow x = \frac{\pi}{8}$$

$$\Rightarrow \tan^{-1}(\tan \frac{9\pi}{8}) = \frac{\pi}{8} \text{ Ans}$$

Exercise 2.1 Q.8ii) Find the domain of the function $\sec^{-1}(2x-3)$

We know that domain of $\sec^{-1}(2x-3)$ is $-1 > 2x-3 > 1$

$$\Rightarrow -1 + 3 > (2x-3) + 3 > 1 + 3$$

$$\Rightarrow 2 > 2x > 4$$

$$\Rightarrow 1 > x > 2$$

$$(-\infty, 1] \cup [2, \infty) \text{ Ans}$$

Exercise 2.1. Q.9i) write the range of branch of $\cos^{-1}x$ other than principal branch.

$$[\pi, 2\pi] \text{ Ans}$$

Exercise 2.1. Q.15.i) Using principal values , find the values of $\cos^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\left(-\frac{1}{2}\right)$

Let $\cos^{-1}\left(\frac{1}{2}\right) = x$, $0 \leq x \leq \pi$

$$\Rightarrow \cos x = \frac{1}{2} = \cos \frac{\pi}{3} \Rightarrow x = \frac{\pi}{3}$$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = x$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$$\Rightarrow \sin x = -\frac{1}{2} = -\sin \frac{\pi}{6} \Rightarrow x = \sin\left(-\frac{\pi}{6}\right) \Rightarrow x = -\frac{\pi}{6}$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{3} - 2\left(-\frac{\pi}{6}\right)$$

$$\Rightarrow \frac{\pi}{3} + \left(\frac{2\pi}{6}\right) \Rightarrow \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3} \text{ ans}$$

Homework Exercise 8.1

(Q.1iii), Q.2iii), Q.5.iii), Q.7i), Q8i), Q.11,Q12.iii) Q13iii), Q14.ii),Q15.iv)

You will find the solutions of following questions in the video link provided to you by school with this assignment,

Q4ii), Q7iii), Q12. i), ii). Q13.ii)

